Return Models
Valuation Multiples For Non-Banks

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In this white paper we will develop the mathematics for valuation multiples for non-banks using the return model discussed in previous sections. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

The table below presents ABC Company's go-forward model assumptions...

Table 1: Model Parameters

Symbol	Description	Balance
R_0	Annualized operating revenue at time zero	800,000
θ	Ratio of operating assets to annualized operating revenue	1.2500
π	Continuous-time after-tax return on assets	0.1450
μ	Continuous-time revenue growth rate	0.0500
κ	Continuous-time weighted-average cost of capital	0.1000

We are tasked with answering the following questions:

Question 1: What is ABC Company's enterprise value as a multiple of operating revenue?
Question 2: What is ABC Company's enterprise value as a multiple of operating assets?
Question 3: What is ABC Company's enterprise value as a multiple of earnings?
Question 4: Why is mean-reversion relevant to ABC Company?

Building Our Model

We will define the variable R_t to be annualized operating revenue at time t and the variable μ to be the continuous-time revenue growth rate. Using the model parameters in Table 1 above, the equation for annualized revenue is...

$$R_t = R_0 \,\text{Exp}\left\{\mu\, t\right\} \tag{1}$$

We will define the variable A_t to be total operating assets at time t and the variable θ to be the ratio of operating assets to annualized operating revenue. Using Equation (1) above and the model parameters in Table 1 above, the equation for assets is...

$$A_t = \theta\, R_t = \theta\, R_0 \,\text{Exp}\left\{\mu\, t\right\} \tag{2}$$

The equation for the derivative of Equation (2) above with respect to time is...

$$\frac{\delta}{\delta t} A_t = \mu\, \theta\, R_0 \,\text{Exp}\left\{\mu\, t\right\} \;\text{...such that...}\; \delta A_t = \mu\, \theta\, R_0 \,\text{Exp}\left\{\mu\, t\right\} \delta t \tag{3}$$

We will define the variable N_t to be annualized after-tax net income at time t and the variable π to be the after-tax return on assets. Using Equation (2) above, the equation for annualized net income is...

$$N_t = \pi\, A_t = \pi\, \theta\, R_t = \pi\, \theta\, R_0 \,\text{Exp}\left\{\mu\, t\right\} \;\text{...where...}\; N_0 = \pi\, \theta\, R_0 \tag{4}$$

We will define the variable V_0 to be enterprise value at time zero and the variable κ to be the continuous-time discount rate. We will define cash flow to be net income (profitability) minus the change in assets (investment). Using Equations (3) and (4) above, the equation for enterprise value is...

$$V_0 = \int_0^{\infty} \Big(N_t \, \delta t - \delta A_t \Big) \text{Exp}\Big\{ -\kappa \, t \Big\} \, \delta t \tag{5}$$

Using Equations (3) and (4) above, we can rewrite Equation (5) above as...

$$\begin{aligned} V_0 &= \int_0^{\infty} \Big(\pi \, \theta \, R_0 \, \text{Exp}\Big\{ \mu \, t \Big\} - \mu \, \theta \, R_0 \, \text{Exp}\Big\{ \mu \, t \Big\} \Big) \text{Exp}\Big\{ -\kappa \, t \Big\} \delta t \\ &= \theta \, R_0 \int_0^{\infty} (\pi - \mu) \, \text{Exp}\Big\{ \mu \, t \Big\} \, \text{Exp}\Big\{ -\kappa \, t \Big\} \delta t \\ &= \theta \, (\pi - \mu) \, R_0 \int_0^{\infty} \text{Exp}\Big\{ (\kappa - \mu) \, t \Big\} \, \delta t \end{aligned} \tag{6}$$

The solution to Equation (6) above is...

$$\begin{aligned} V_0 &= \theta \left(\frac{\pi - \mu}{\mu - \kappa} \right) R_0 \left(\text{Exp}\Big\{ (\kappa - \mu) \times \infty \Big\} - \text{Exp}\Big\{ (\kappa - \mu) \times 0 \Big\} \right) \\ &= \theta \left(\frac{\pi - \mu}{\mu - \kappa} \right) R_0 \Big(0 - 1 \Big) \\ &= \theta \left(\frac{\pi - \mu}{\kappa - \mu} \right) R_0 \end{aligned} \tag{7}$$

Using Equations (1), (2), and (4) above, we can rewrite Equation (7) above as...

$$V_0 = \theta \left(\frac{\pi - \mu}{\kappa - \mu} \right) R_0 = \left(\frac{\pi - \mu}{\kappa - \mu} \right) A_0 = \frac{1}{\pi} \left(\frac{\pi - \mu}{\kappa - \mu} \right) N_0 \tag{8}$$

Using Equation (8) above, our three enterprise value valuation multiples are...

$$\text{Assets} = \frac{\pi - \mu}{\kappa - \mu} \;\; \text{...and...} \;\; \text{Revenue} = \frac{\theta \, (\pi - \mu)}{\kappa - \mu} \;\; \text{...and...} \;\; \text{Earnings} = \frac{1}{\pi} \left(\frac{\pi - \mu}{\kappa - \mu} \right) \tag{9}$$

Answers To Our Hypothetical Problem

Question 1: What is ABC Company's enterprise value as a multiple of operating revenue?

Using Equation (9) above and the data in Table 1 above, our valuation multiple as a function of annualized revenue is...

$$\text{Multiple of annualized operating revenue} = \frac{\theta \, (\pi - \mu)}{\kappa - \mu} = \frac{1.2500 \times \, (0.1450 - 0.0500)}{0.1000 - 0.0500} = 2.3750 \tag{10}$$

Using Equation (10) above, the equation for enterprise value as a function of operating revenue is...

$$V_0 = 2.3750 \times 800,000 = 1,900,000 \;\; \text{...where...} \;\; \text{Revenue} = R_0 = 800,000 \tag{11}$$

Question 2: What is ABC Company's enterprise value as a multiple of operating assets?

Using Equation (9) above and the data in Table 1 above, our valuation multiple as a function of operating assets is...

$$\text{Multiple of operating assets} = \frac{\pi - \mu}{\kappa - \mu} = \frac{0.1450 - 0.0500}{0.1000 - 0.0500} = 1.9000 \tag{12}$$

Using Equation (12) above, the equation for enterprise value as a function of operating assets is...

$$V_0 = 1.9000 \times 1,000,000 = 1,900,000 \text{ ...where... Assets} = \theta R_0 = 1.2500 \times 800,000 = 1,000,000 \quad (13)$$

Question 3: What is ABC Company's enterprise value as a multiple of earnings? Using Equation (9) above and the data in Table 1 above, our valuation multiple as a function of earnings is...

$$\text{Multiple of annualized earnings} = \frac{1}{\pi}\left(\frac{\pi - \mu}{\kappa - \mu}\right) = \frac{1}{0.1450} \times \frac{0.1450 - 0.0500}{0.1000 - 0.0500} = 13.1034 \quad (14)$$

Using Equation (14) above, the equation for enterprise value as a function of annualized earnings is...

$$V_0 = 13.1034 \times 145,000 = 1,900,000 \text{ ...where... Earnings} = \pi\,\theta\,R_0 = 0.1450 \times 1.2500 \times 800,000 = 145,000 \quad (15)$$

Question 4: Why is mean-reversion relevant to ABC Company?

ABC Company's return on assets is 14.50% and its weighted-average cost of capital is 10.00%. In the analysis above we are assuming that the return on assets and cost of capital are static in perpetuity. In competitive economies the return on assets should mean-revert to the cost of capital over time. We are assuming that this mean reversion never happens. Note that if ABC Company's return on assets equaled its cost of capital then enterprise value would decrease from $1,900,000 to $1,000,000.